

Thermodynamic analysis of a pulse tube engine

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ABSTRACT

The pulse tube engine is an innovative simple heat engine based on the pulse tube process used in cryogenic cooling applications. The working principle involves the conversion of applied heat energy into mechanical power, thereby enabling it to be used for electrical power generation. Furthermore, this device offers an opportunity for its wide use in energy harvesting and waste heat recovery. A numerical model has been developed to study the thermodynamic cycle and thereby help to design an experimental engine. Using the object-oriented modeling language Modelica, the engine was divided into components on which the conservation equations for mass, momentum and energy were applied. These components were linked via exchanged mass and enthalpy. The resulting differential equations for the thermodynamic properties were integrated numerically. The model was validated using the measured performance of a pulse tube engine. The transient behavior of the pulse tube engine's underlying thermodynamic properties could be evaluated and studied under different operating conditions. The model was used to explore the pulse tube engine process and investigate the influence of design parameters.

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1. Introduction

Lord Rayleigh discovered that an axial temperature gradient along a tube can amplify acoustical oscillations. In the inverse case acoustical oscillation can be applied to a tube with the result of an axial temperature gradient. In both cases a heat flux is converted into a work flux and vice versa. The underlying phenomena of thermoacoustic oscillations was elaborated by Rott [1]. Engines using this phenomena are known as thermoacoustic heat engines or thermoacoustic refrigerators. Since the late 1970s intensive investigations were made by Ceperly [2], Wheatley et al. [3] and Backhaus and Swift [4] to study and develop thermoacoustic prime movers. Due to the employment of acoustics waves, these engines need large resonators for their operation. Since pressure swing and power density are small, they have never reached marketability.

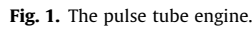
A process similar to thermoacoustic processes, but with higher pressure amplitudes and thus higher power output is the pulse tube process presented in 1964 by Gifford and Longworth [5]. The pulse tube refrigerator has been investigated and developed for about 40 years and is now employed in cryogenic technologies to provide cooling power at very low temperatures. In accordance with thermoacoustics, the pulse tube process uses a phase shift between gas movement and heat transfer to create a thermodynamic cycle.

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Reversing, the pulse tube process allows for the design of a simple prime mover for the conversion of externally supplied thermal power into mechanical power and respectively, into electrical power by using a generator. Prior concepts using the pulse tube principle to design a heat engine were presented in 1987 by Chen and West [6]. In 1995 Taler was granted a US patent [7] for a heat engine based on a phase delay between pressure fluctuations and heat transfer (thermal lag). The concept of the pulse tube engine was finally presented by Hamaguchi et al. [8] in 2005 and Organ [9] in 2007. A number of experimental investigations to characterize this engine were also published by Hamaguchi et al. [8,10]. According to work flux measurements of Yoshida et al. [11] in 2009, the pulse tube engine can be classified into the thermoacoustic standing wave engine group with an underlying intrinsically irreversible thermodynamic cycle. Nevertheless, due to its simplicity, the pulse tube engine has potential for the usage of low grade waste heat, e.g. of internal combustion engines [12].

The basic design of the pulse tube engine is given in Fig. 1. The heat input is supplied via a hot heat exchanger (h_{hx}). A thermal buffer (pulse tube) is joined to the left side of the hot heat exchanger. For heat emission a cold heat exchanger (ch_{x1}) is situated at the pulse tube's cold side. The pulse tube should have almost adiabatic behavior. A device to store heat (regenerator) with a second cold heat exchanger (ch_{x2}) at its cold side to set the shown material temperature distribution is joined to the right side of the hot heat exchanger. The regenerator should have perfect isothermal behavior. The conversion of compression/expansion work into mechanical work and vice versa is carried out by the resulting pressure



The depicted material temperature distribution – as a property of the pulse tube and the regenerator – results from the layout of the heat exchangers. This, in combination with the heat transfer conditions, leads to a thermodynamic asymmetry between pulse tube and regenerator. The functional principle of the pulse tube engine is based on the simultaneous compression and moving of the working gas into an area of high material temperature, followed by a simultaneous expansion and moving of the working gas into an area of low material temperature. The thermodynamic asymmetry prevents the working gas from being heated during compression, although it enables it to be heated during expansion. Therefore, it becomes possible to take up heat at high pressure and give off heat at low pressure.

In order to formulate the governing equations of the pulse tube engine process, the following simplifications were made:

- Fig. 2.** Model of the pulse tube engine.

- Eqs. (7) and (8) are applied to each component. In the ideal model heat fluxes $\dot{Q}$ exclusively occur due to heat transfer between the working gas and the surrounding material.

2.2. Cylinder

The volume and the heat flux coefficient are given by

$$V(t) = V_0 + \frac{V_d}{2} \left(1 + r_{cr} - \cos(\omega t - \phi) - \sqrt{r_{cr}^2 - \sin^2(\omega t - \phi)} \right), \quad (9)$$

and

$$\gamma(t) = \gamma_0 + \frac{\gamma_d}{2} \left(1 + r_{cr} - \cos(\omega t - \phi) - \sqrt{r_{cr}^2 - \sin^2(\omega t - \phi)} \right), \quad (10)$$

where the crank ratio $r_{cr} = 2L_r/s$ is the ratio between piston rod length L_r and crank radius $s/2$, which is half of the stroke s . The cycle frequency is defined by $\omega = 2\pi f$ and the starting position of the piston is given by ϕ , whereas $\phi = 0$ at top dead center (TDC). In Eq. (9) V_0 and V_d are the dead volume and the displacement volume and in Eq. (10) γ_0 and γ_d are components of the heat flux coefficient. The compression/expansion power and the amount of heat exchanged between the working gas and the surrounding material with the material temperature T_m are

$$\dot{W} = -p \frac{dV}{dt} \quad \text{and} \quad \dot{Q} = \gamma(T_m - T). \quad (11)$$

2.3. Heat exchangers

With the fluxes $\dot{m}_L$ and $\dot{H}_L$ on the left side (L) and the fluxes $\dot{m}_R$ and $\dot{H}_R$ on the right side (R) of each component, Eqs. (7) and (8) reduce to

$$\frac{dp}{dt} = \frac{R_s T}{V} (\dot{m}_L + \dot{m}_R), \quad (12)$$

and

$$0 = -c_v T (\dot{m}_L + \dot{m}_R) + \dot{Q} + \dot{H}_L + \dot{H}_R, \quad (13)$$

where the temperature of the cold heat exchangers at any time is $T = T_c$ and of the hot heat exchanger $T = T_h$, respectively. The heat flux between gas and surrounding material results from Eq. (13).

2.4. Pulse tube and regenerator

Both components have constant volume. As they are located between hot and cold heat exchangers, they have a strong axial temperature distribution. To take this characteristic into account, the regenerator and the pulse tube are each divided into n subvolumes

with a characteristic temperature T_i , shown in Fig. 3. Each subvolume has a time-invariant material temperature $T_{m,i}$, which can be expressed for the pulse tube by

$$T_{m,i}^{pt} = T_c + \frac{i-1}{n-1} (T_h - T_c), \quad (14)$$

and for the regenerator by

$$T_{m,i}^{reg} = T_h - \frac{i-1}{n-1} (T_h - T_c). \quad (15)$$

The temperature distributions are symmetric to the hot heat exchanger. According to Fig. 3 Eqs. (7) and (8) were applied at each subvolume i . This leads to

$$\frac{dp}{dt} = \frac{R_s T_i}{V/n} (\dot{m}_{i-1} + \dot{m}_i) + \frac{p}{T_i} \frac{dT_i}{dt}, \quad (16)$$

and

$$c_v \frac{pV/n}{R_s T_i} \frac{dT_i}{dt} = -c_v T_i (\dot{m}_{i-1} + \dot{m}_i) + \dot{Q}_i + \dot{H}_{i-1} + \dot{H}_i. \quad (17)$$

These equations were applied to the pulse tube. The heat flux between gas and surrounding pulse tube material is given by

$$\dot{Q}_i = \frac{\gamma}{n} (T_{m,i} - T_i). \quad (18)$$

Since the subvolumes of the regenerator are characterized by isothermal behavior, Eqs. (16) and (17) simplify for the regenerator to

$$\frac{dp}{dt} = \frac{R_s T_i}{V/n} (\dot{m}_{i-1} + \dot{m}_i), \quad (19)$$

and

$$0 = -c_v T_i (\dot{m}_{i-1} + \dot{m}_i) + \dot{Q}_i + \dot{H}_{i-1} + \dot{H}_i. \quad (20)$$

In analogy to the heat exchangers, the heat flux between gas and surrounding material results from Eq. (20).

The boundary conditions for pulse tube and regenerator are $\dot{m}_{i-1} = \dot{m}_L$ and $\dot{H}_{i-1} = \dot{H}_L$ for $i = 1$ (left side) as well as $\dot{m}_i = \dot{m}_R$ and $\dot{H}_i = \dot{H}_R$ for $i = n$ (right side). The total heat transfer is calculated with $\dot{Q}_t = \sum_{i=1}^n \dot{Q}_i$.

Linking two control volumes A and B is achieved by exchanging mass and enthalpy. From Kirchhoff's laws it can be derived that the fluxes add up ($\dot{m}_A + \dot{m}_B = 0$ and $\dot{H}_A + \dot{H}_B = 0$) while the pressure remains equal ($p_A = p_B$). In order to determine the magnitude of the enthalpy fluxes the following consideration was made: When the mass $\dot{m}_A$ flows from control volume A to control volume B it transports the enthalpy $\dot{H}_A$ from control volume A to control volume B and vice versa. Depending on the direction of the mass flux, the temperature to calculate the enthalpy flux has to be switched hardly between the temperatures of control volume A and B. To handle numerical instabilities resulting from this process, the temperature for calculating the enthalpy flux was modified by $T^* = \lambda T_A + (1 - \lambda) T_B$, where a smoothing function

$$\lambda = \begin{cases} 1, & \dot{m}_A > \dot{m}_{tr} \\ \frac{\dot{m}_A}{2\dot{m}_{tr}} + \frac{1}{2}, & -\dot{m}_{tr} \leq \dot{m}_A \leq \dot{m}_{tr} \\ 0, & \dot{m}_A < -\dot{m}_{tr} \end{cases}, \quad (21)$$

with a threshold value $\dot{m}_{tr}$ for the mass flux was introduced. With the specific heat capacity at constant pressure c_p the enthalpy flux can then be calculated according to $\dot{H}_A = -\dot{H}_B = \dot{m}_A c_p T^*$. The gross power P_{gross} , respectively, the transferred cycle heat fluxes $\dot{Q}_{cyc}$ within the components can be calculated by integrating over one cycle and multiplying by the operating frequency $P_{gross} = f \oint \dot{W} dt$ and $\dot{Q}_{cyc} = f \oint \dot{Q} dt$, respectively. This allows the computation of all cycle energy fluxes.

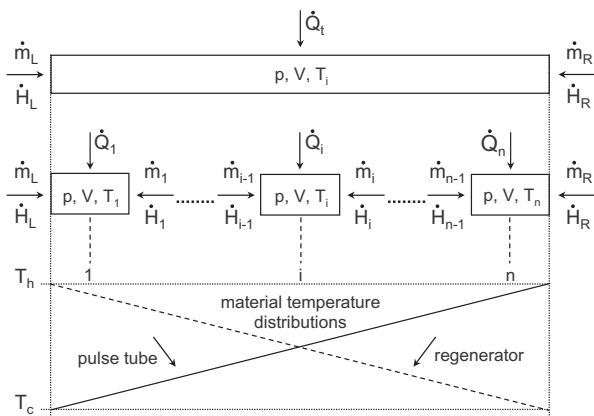


Fig. 3. Pulse tube and regenerator.

2.5. Computation parameters

The solution of the system of differential equations was obtained numerically by implementing the equations in the object-oriented programming language Modelica [14]. The time related integration was made with an implicit procedure of variable order and variable increments. The maximum permissible time increment was set at a value of $dt_{max} = 10^{-5}$ s.

Since pulse tube and regenerator consist of a number of subvolumes n_{pt} and n_{reg} , the dependency of the solution on n was investigated. For all calculations both values were set to $n_{pt} = n_{reg} = n$. Fig. 4 shows the calculated cycle values of the compression/expansion work W_{gross} as well as of the transferred heat in the pulse tube Q_{pt} and in the regenerator Q_{reg} depending on n . The calculations were done for a representative test engine of 100 cm³ displacement volume, filled with air (pressure: 100 kPa) at bottom dead center (BDC), an operating frequency of 5 Hz and minimum/maximum temperatures of $T_c = 27$ °C and $T_h = 327$ °C. The cylinder was regarded as adiabatic and the heat transfer coefficient in the pulse tube was set to $\gamma_{pt} = 0.2$ W/K. The order of magnitude of the mass fluxes is 1 g/s. The threshold value in the smoothing function (21) was set two orders of magnitude smaller to a value of $\dot{m}_{tr} = 10$ mg/s. Therefore, the influence on the solution is small. The dependency of all energy values on the number of subvolumes is clearly evident. For high values of n the results converge towards threshold values. The transferred heat in the regenerator has the highest dependency. Its values were adapted with the function $Q_{reg}(n) = (a + b n)^{-1}$, since the transferred heat of an ideal regenerator over one cycle equals zero. From Fig. 4 can be derived, that a preferably large number of subvolumes should be selected. This results in a high computational demand. Following standard practice in technology, a compromise between accuracy and effort was chosen. For all calculations the number of subvolumes was set to $n = 100$. Choosing this value the inaccuracy of the calculated regenerator's transferred heat over one cycle amounts to 0.15 J which is about 1% of the regenerator's stored heat of 14.9 J during compression.

2.6. Power losses

To make it possible to compare the results of the ideal model with the measured performance of a pulse tube engine, the calculated gross power P_{gross} has to be corrected by power losses P_{loss} . Four main loss mechanism were regarded:

- (1) Heat conduction through pulse tube and regenerator.
- (2) Imperfect heat transfer in the heat exchangers and the regenerator.

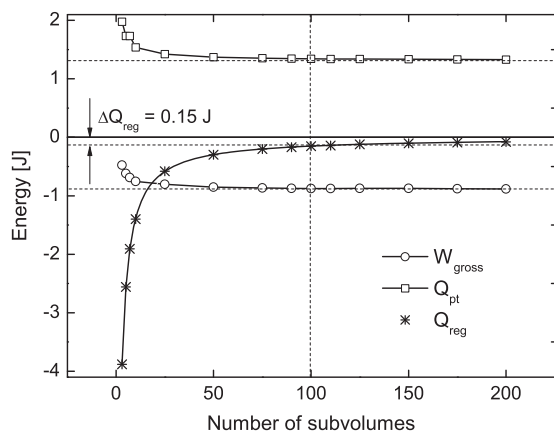


Fig. 4. Influence of the number of subvolumes.

- (3) Friction and pressure drop in the heat exchangers and the regenerator.
- (4) Friction between piston and cylinder and in the bearings of the crank mechanism.

In the heat exchangers, the transferred heat is calculated using fixed heat exchanger temperatures. Thus, only heat conduction through the gas has an influence on the cycle power. Both the conductive heat flux through the pulse tube and through the regenerator can be calculated by $\dot{Q}_{cd} = \lambda A_f (T_h - T_c) / L$, where λ is the mean heat conductivity of the working fluid, A_f the free flow area, and L the length of the regarded component. The resulting power loss due to heat conduction via the working gas is obtained by multiplying the conductive heat fluxes by the engine's thermal efficiency

$$P_{loss}^{cd} = \eta_{th} (Q_{cd}^{pt} + Q_{cd}^{reg}). \quad (22)$$

Imperfect heat transfer in the heat exchangers can be considered using a heat exchanger efficiency $\eta_{hx} = \dot{Q}_{re} / \dot{Q}_{id}$, where $\dot{Q}_{id}$ and $\dot{Q}_{re}$ are the maximal possible and the real heat flux. Typical values for heat exchangers are $\eta_{hx} = 0.7 \dots 0.95$ [15]. Due to their high heat transfer area, in general, regenerators have very high values of $\eta_{hx} > 0.99$ [16]. Thus, the regenerator's heat transfer loss is negligible. To quantify the influence of the heat exchanger efficiency, the global energy balance on the entire engine was used. The power of the ideal cycle is given by $P_{id} = -\sum_i \dot{Q}_{id,i}$. With the heat exchanger efficiency the power of the real cycle is given by $P_{re} = -\sum_i \dot{Q}_{re,i} = -\eta_{hx} \sum_i \dot{Q}_{id,i}$. Thus, the power loss due to imperfect heat transfer in the heat exchangers amounts to

$$P_{loss}^{hx} = |P_{id} - P_{re}| = (1 - \eta_{hx}) |P_{id}|. \quad (23)$$

The friction force resulting from the mass flux $\dot{m}$ flowing through the heat exchangers and the regenerator leads to the power loss due to gas friction represented by

$$P_{loss}^{fg} = f \oint \frac{2f_f |\dot{m}|^2 L}{\rho^2 A_f^2 d_h} dt, \quad (24)$$

with the friction factor f_f , the gas density ρ and the hydraulic diameter d_h . The mass flow rate and the gas density are results of the ideal transient simulation. Since the flow paths in the heat exchangers are tight, the value of the Reynolds number $Re \propto d_h$ is small and the friction factor in the heat exchangers can be expressed by $f_f(Re) = 16/Re$, which is valid for laminar flows. Assuming that the regenerator is made of stacked wire mesh screens with the porosity e , the friction factor $f_f(Re, e)$ can be calculated using tables from Ref. [17].

The total internal power loss is given by

$$P_{loss}^{int} = P_{loss}^{cd} + P_{loss}^{hx} + P_{loss}^{fg}, \quad (25)$$

and the net power can be calculated with

$$P_{net} = P_{gross} + P_{loss}^{int}. \quad (26)$$

To get the shaft power, the net power has to be reduced by external power loss in the cylinder and in the crank mechanism. These power losses were calculated from the friction force between the piston and the cylinder F_f^cy and the friction moments $M_{f,i}^{cr}$ in the connecting rod and the crankshaft bearings. The resulting power loss due to mechanical friction is

$$P_{loss}^{ext} = f \oint \left(|F_f^cy u_p| + \sum_i |M_{f,i}^{cr} \omega_i| \right) dt, \quad (27)$$

with the piston velocity u_p and the angular velocity ω_i of the bearings i . The shaft power is obtained from

$$P_{shaft} = P_{net} + P_{loss}^{ext}. \quad (28)$$

All power outputs have a negative and all power losses a positive sign.

3. Reference engine

For all investigations the parameters given in Table 1 according to the pulse tube engine reported in [8], pages 6–11, were used. The threshold value in the smoothing function (21) was set to $m_{tr} = 10$ mg/s. The working fluid is air, which is regarded as a perfect gas. The temperature T_{c1} was obtained directly from the author of Hamaguchi et al. [8]. The temperatures T_{c2} and T_h were taken from Ref. [8], page 10, Fig. 14, measured at an operating frequency of 2.5 Hz. Nevertheless, these temperatures were used for all frequencies. The pulse tube and cylinder heat flux coefficients were estimated using heat transfer correlations from Ref. [18].

Since the hot heat exchanger is part of the regenerator and the cold heat exchanger 2 is a solid block of aluminum, their dead volumes are zero. In contrast to that, the remaining dead volumes of the cylinder and cold heat exchanger 1 are unknown and needed to be determined. To do so, the dead volume V_0 was introduced and assigned completely to the cylinder while the dead volume of cold heat exchanger 1 was considered as zero. The error of this approach is small, because the time-averaged temperatures in both components have similar values. For an operating frequency of 5 Hz the dead volume V_0 and the filling pressure p_0 at BDC were varied to meet the measured minimal pressure of $p_{min} = 87.1$ kPa and the measured pressure amplitude of $\Delta p = 28.4$ kPa according to Hamaguchi et al. [8], page 10, Fig. 13. The results are $V_0 = 48$ cm³ and $p_0 = 88.4$ kPa, respectively. The found high value of the dead volume could be the result of leakage occurring in the engine, which lowers the pressure swing intensity. In this case V_0 represents a combination of the real unknown dead volume with virtual dead volume due to leakage.

To take power losses into account, the following parameters were used:

- (1) Mean heat conductivity: 0.035 W/(m²K).
- (2) Heat exchanger efficiency: $\eta_{hx} = 0.9$.
- (3) Mean diameter of all bearings: $d_b = 5$ mm.
- (4) Mechanical friction factor (steel–steel, lubricated): $f_m = 0.01$.

Two design parameters were defined. The distribution ratio is the ratio of regenerator volume V_{reg} to pulse tube volume V_{pt}

Table 1
Reference engine parameters according to Hamaguchi et al. [8].

Parameter	Symbol	Value
Temperature h _{hx}	T_h	332 °C
Temperature ch _{x1}	T_{c1}	35 °C
Temperature ch _{x2}	T_{c2}	51 °C
Ambient pressure	p_a	101.325 kPa
Filling pressure @ BDC	p_0	88.4 kPa
Minimum pressure	p_{min}	87.1 kPa
Pressure amplitude	Δp	28.4 kPa
Cylinder bore	b	27.6 mm
Piston stroke	s	60.0 mm
Crank ratio	r_{cr}	2
Pulse tube diameter	d_{pt}	27.2 mm
Pulse tube length	L_{pt}	73.5 mm
Regenerator diameter	d_{reg}	27.2 mm
Regenerator length	L_{reg}	91.0 mm
Mesh number	n_m	50
Wire diameter	d_w	193 μm
Porosity	e	0.762
Displacement volume	V_d	35.90 cm ³
Dead volume	V_0	48.00 cm ³
Pulse tube volume	V_{pt}	42.71 cm ³
Regenerator volume	V_{reg}	40.29 cm ³
Cylinder heat flux coeff.	γ_{cyl}	0.16 W/K
Pulse tube heat flux coeff.	γ_{pt}	0.19 W/K

$$r_d := \frac{V_{reg}}{V_{pt}}, \quad (29)$$

and the compression ratio was defined as

$$r_c := \frac{V_{pt} + V_{reg} + V_{hx} + V_d}{V_{pt} + V_{reg} + V_{hx}} = 1 + \frac{V_d}{V_{pt} + V_{reg} + V_{hx}}, \quad (30)$$

where V_d represents the displacement volume of the cylinder and V_{hx} the sum of the heat exchanger volumes. The dead volume V_0 was not taken into account. With the assumption of $V_{hx} = 0$, the regarded engine has a distribution ratio of $r_d = 0.94$ and a compression ratio of $r_c = 1.43$.

4. Model validation

To validate the developed simulation model, the measured characteristic of the reference engine presented in Section 3 was used.

In Fig. 5 the calculated pressure and the volume displacement of the cylinder over one engine cycle with an operating frequency of 5 Hz are shown. The ambient pressure of $p_a = 101.325$ kPa (standard conditions at sea level) is reached at about 75 deg before TDC. Initializing the engine process with ambient pressure at about 85 deg before TDC leads to similar results. Thus, in the experiment, the engine was started around this position of the piston. The phase difference between TDC and maximum pressure is 1.96 deg. The measured value given in [8], page 10, is 2.28 deg. The mismatch amounts to 14%.

Fig. 6 shows the resulting p–V-diagram. The enclosed area represents the gross indicated work, which amounts to $W_{gross} = -32.1$ mJ. This value can only be reached under ideal conditions. In the real engine the shape of the pressure curve is significantly influenced by loss mechanisms such as heat conduction, heat transfer losses and pressure drop (see Section 2.6). The last one mainly occurs in the regenerator and cold heat exchanger 1. Taking the mentioned energy dissipations of about 6.8 mJ into account, the net indicated work amounts to -25.2 mJ. The measured value in [8], page 8, is -24.4 mJ. The difference amounts to 3.5%. The estimated work dissipation due to mechanical friction in the cylinder and the crank mechanism is 2.6 mJ. The resulting shaft work is -21.6 mJ. In [8], page 10, Fig. 11, a measured value of -19 mJ is reported. The mismatch of about 14%, compared to the good match of the net indicated work is due to the high uncertainty of the calculated mechanical friction loss mechanisms.

In Fig. 7 the frequency dependency of the absolute values of the calculated net indicated power P_{net} and the calculated shaft power P_{shaft} is compared with the measured values of Hamaguchi et al. [8], page 10, Fig. 11. The order of magnitude of both calculated power

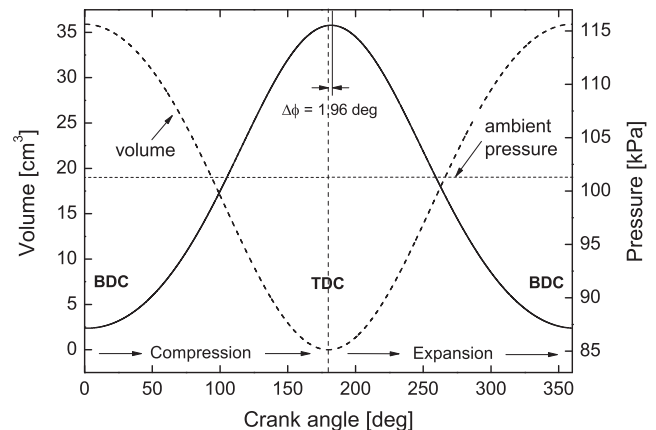


Fig. 5. Relationship between volume displacement and working gas pressure.

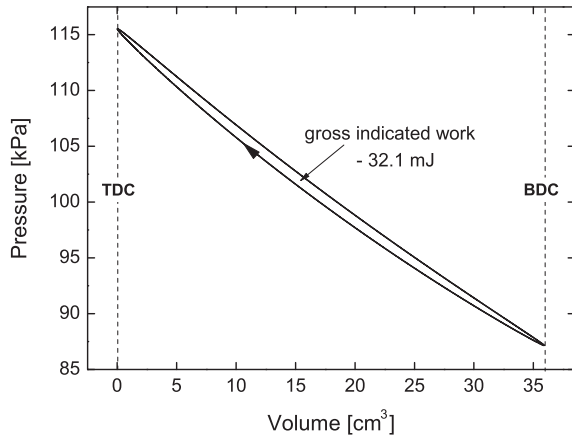


Fig. 6. p-V-diagram.

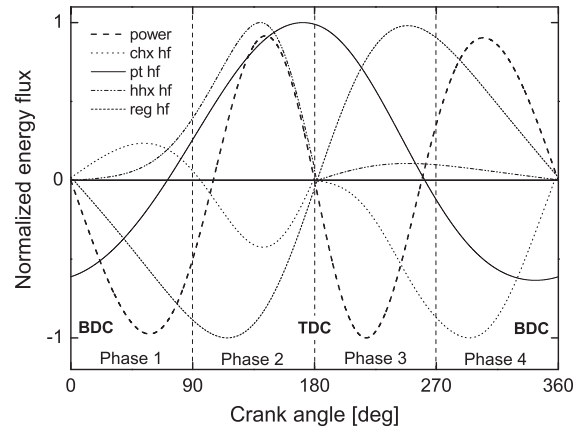


Fig. 8. Energy fluxes in the engine components.

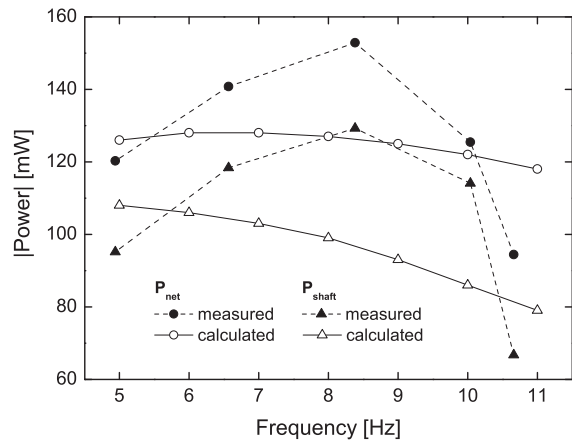


Fig. 7. Comparison of measured and calculated power outputs.

outputs is in good agreement with the measurements. The property of a maximum value for a certain frequency is clearly evident for measurement and calculation. Compared to the calculated values, the measured power values have a much stronger dependency on the operating frequency. Several reasons are responsible for this behavior. The strong increase of the measured power outputs for an increasing operating frequency between 5 Hz and 8 Hz could be a result of lower leakage due to higher gap flow friction of the gas leaking through the gap between piston and cylinder. Therefore, for higher frequencies, the engine probably is getting better sealed. The heat exchanger temperatures, the minimum pressure and the pressure amplitude were taken from measurements at low operating frequencies and used for the calculations at higher frequencies. Thus, the real pressure curve and the transferred heat for higher operating frequencies can not be clearly reproduced by the model. Moreover, the loss mechanisms – in reality – interact. A pressure drop in the regenerator reduces the mass flux and thus, reduces the transferred heat per time. This reduces the power output stronger than calculated. The strong non-linear behavior of the engine, especially at high operating frequencies, can not be reproduced with the presented ideal model.

5. Simulation results

After validation the simulation model was used to investigate the pulse tube engine process and study the influence of design parameters on the engine performance.

5.1. Pulse tube engine process

Based on the results of the validation, the model can be used to study the pulse tube engine process at an operating frequency of 5 Hz. At this frequency the modeling error is smaller than 15%.

Referring to Fig. 5 two results are evident. First, the working gas pressure falls during the second phase of the expansion below ambient pressure. Thus, the surrounding air is able to carry out work on the system due to the resulting force on the piston. This slows down the second phase of the expansion while assisting the first phase of the compression, which provides for a free piston operation of the engine. Second, the positive phase delay of 1.96 deg between pressure and TDC results in a slightly higher expansion pressure than compression pressure and is therefore responsible for a release of work.

Fig. 8 shows the energy fluxes occurring in the components of the engine as well as the compression/expansion power difference, which is calculated from the pressure difference between the engine and the ambient during each phase of the process. The shaft power is proportional to the calculated compression/expansion power difference. The operating conditions are the same as in Fig. 5. The energy fluxes were normalized to their absolute maximums. A positive sign means that the energy flux is added to the working gas. In phases 1 and 2 the piston moves from BDC to TDC, resulting in a compression of the working gas. During phases 3 and 4 the piston moves from TDC to BDC, which results in an expansion of the working gas. Therefore the process cycle can be described as follows:

5.1.1. Phase 1: compression

Since the ambient pressure is higher than the engine pressure, the ambient air pushes the piston in the direction of TDC, which is associated with a shaft power output provided by the ambient air. The working gas is compressed. Simultaneously, it receives heat from the cold heat exchanger 1 (chx hf), located between the cylinder and the pulse tube. Therefore it must have gone below T_{c1} during the previous expansion process. A heat input by the hot heat exchanger (hhx hf) starts at about mid-phase. Within the pulse tube the direction of the heat flux (pt hf) changes from heat output to heat input. The regenerator material absorbs heat (reg hf).

5.1.2. Phase 2: compression

At the beginning of this phase the ambient air is still providing a shaft power output. At about 100 deg after BDC the engine pressure exceeds the ambient pressure and compression power has to be delivered to the shaft in order to further compress the working

gas. At the beginning of this phase, the direction of the heat flux in the cold heat exchanger 1 reverses and the compressed gas is cooled on its way to the pulse tube. In the pulse tube and the hot heat exchanger, an enforced addition of heat to the working gas takes place. The regenerator material still absorbs heat.

5.1.3. Phase 3: expansion

For the majority of this phase the engine pressure is higher than the ambient pressure. The working gas is expanding and pushes the piston in the direction of BDC, associated with a shaft power output provided by the working gas. At about 80 deg after TDC the engine pressure falls below the ambient pressure and expansion power has to be delivered to the shaft in order to continue the expansion of the working gas. The regenerator material gives off heat. The heat flux in the hot heat exchanger comes to an almost complete standstill since the small temperature difference between the working gas, coming from the regenerator, and the heat exchanger material allows for almost no heat to be supplied to the gas. Within the pulse tube the direction of the heat flux changes from heat input to heat output. The cold heat exchanger withdraws heat from the working gas on its way to the cylinder.

5.1.4. Phase 4: expansion

To further expand the working gas, expansion power has to be delivered to the shaft. The regenerator material still gives off heat and the heat flux in the hot heat exchanger is still almost zero. In the pulse tube heat emission takes place and the cold heat exchanger keeps withdrawing heat from the gas.

If the absolute amount of energy added to the shaft is smaller than the absolute amount of energy released from the shaft, the engine acts as a prime mover. The energy delivered to the shaft has to be stored in a flywheel or any other mechanism. In the case of a free piston engine, this could be the kinetic energy of the piston.

5.2. Compression ratio

The influence of the compression ratio was studied. The displacement and the dead volume were fixed at the values given in Table 1. The distribution ratio was kept at $r_d = 0.94$, whereas the sum of pulse tube and regenerator volume was varied. The engine was filled with ambient air ($p_a = 101.325$ kPa) at 85 deg before TDC and run at an operating frequency of 5 Hz.

In Fig. 9 the calculated relationships between the compression ratio and the gross power as well as between the compression ratio and gross efficiency are shown. The engine process was calculated for three temperatures of the hot heat exchanger, which are $T_h = 332$ °C from Table 1 as well as $T_h = 200$ °C and $T_h = 500$ °C. It can be seen, that the performance of the engine strongly depends on the compression ratio. For the gross power and the gross efficiency optimal values exist. They have higher absolute values at higher compression ratios for an increasing temperature of the hot heat exchanger. Turning this result around, leads to the conclusion that in a free piston engine with variable stroke the amplitude of the piston oscillation will increase with an increase of the hot heat exchanger temperature. The engine provides a gross power output even for $T_h = 200$ °C, which indeed is too small to provide a significant shaft power output.

The highest gross efficiency is reached at higher compression ratios than the highest gross power. The difference increases with an increase of the temperature. For $T_h = 200$ °C the difference is nearly zero, for $T_h = 332$ °C it amounts to 0.2 and for $T_h = 500$ °C to 0.4. In the case of $T_h = 332$ °C, the highest gross power of about -0.2 W is reached at a compression ratio of $r_c = 1.9$ and the highest gross efficiency of about 8.2% is reached at a compression ratio of $r_c = 2.1$.

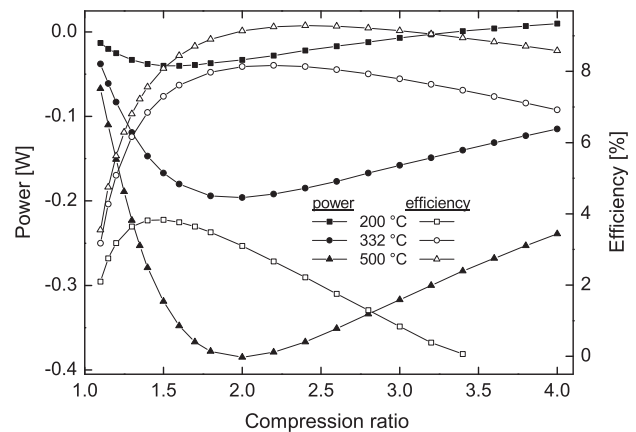


Fig. 9. Relationship between compression ratio and engine performance.

At very low compression ratios the engine does not function, because the pressure difference between compression and expansion is too small. The observed increase of the engine performance up to the optimal compression ratio results from an increasing mass flux passing the hot heat exchanger. The transferred heat and thus the converted energy into expansion work increases. On the other hand, the slope of the transferred heat decreases. This is due to the stronger temperature rise during compression, which lowers the temperature difference between the hot heat exchanger and the working gas. For this reason, the optimal compression ratio increases for higher temperatures of the hot heat exchanger.

At higher compression ratios, the pressure rises stronger and heat is added to the working gas at a higher pressure. Additionally, for lower temperature differences, the entropy production due to heat transfer decreases. This increases the efficiency and is responsible for the higher compression ratio of maximum efficiency compared to the compression ratio of maximum power output. The highest efficiency is reached when the compression end temperature is equal to the temperature of the hot heat exchanger. Due to the resulting lower amount of transferred heat, the power decreases already at lower compression ratios.

At very high compression ratios the temperature rise during compression is too strong, which leads to a lower amount of heat transferred to the working gas passing the hot heat exchanger. Hence, the energy converted into expansion work decreases and leads to a not-functioning engine.

5.3. Distribution ratio

To explore the effect of the distribution ratio, the sum of the pulse tube volume and the regenerator volume was kept constant at $V_{pt} + V_{reg} = 83$ cm³. The distribution ratio was varied between 0.2 and 12 while fixing the lengths and varying the diameters of pulse tube and regenerator. All other dimensions and operating conditions were kept at the values given in Table 1. The operating frequency was 5 Hz.

Fig. 10 shows the gross and shaft power as well as the gross and shaft efficiency of the engine depending on the distribution ratio. All values have a strong dependency on the distribution ratio and come to zero for $r_d \rightarrow \infty$. For very small distribution ratios the engine does not provide a power output. The efficiency has its maximum between $r_d = 1.2$ (gross power) and $r_d = 1.7$ (shaft power). The maximum gross and shaft power are both obtained at $r_d = 4.7$, which can be regarded as a limit of optimal operation. The value of $r_d = 0.94$ used in [8] is therefore far from the optimum value for maximum power output.

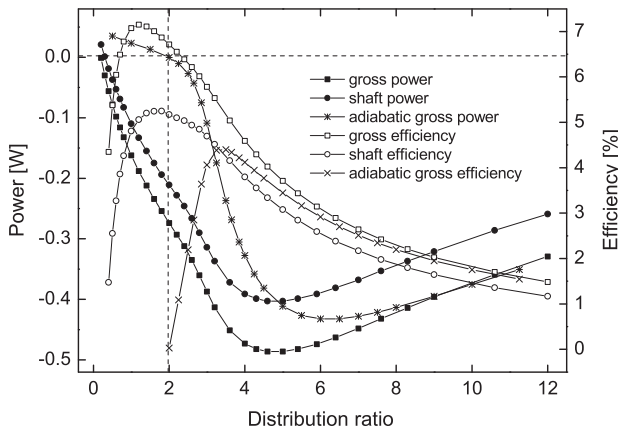


Fig. 10. Relationship between distribution ratio and engine performance.

In [10], pages 1268–1270, a pulse tube engine is presented which has the same regenerator volume as the one presented in [8], but a 22% lower pulse tube volume. The distribution ratio is $r_d = 1.21$ and thus, according to Fig. 10, in the range of the highest efficiency. The measured shaft power is about 50% higher than the shaft power of the engine presented in [8]. Changing the distribution ratio from $r_d = 0.94$ to $r_d = 1.21$, the model predicts a shaft power improvement of only 36%. The mismatch is due to the increased compression ratio of the engine in [10], which was increased from $r_c = 1.43$ to $r_c = 1.47$. This results according to Fig. 9 in an additional increase of the power output of about 12%.

Moreover, in [10], pages 1272–1275, is reported, that additional heat storage material (stacked wire mesh screens: 5 mesh/in., 36 mm in length) placed at the hot side of the pulse tube increases the power output of the engine dramatically of about 2.7 times compared to the version without the additional heat storage material. The created additional heat storage gas volume amounts to about 18 cm³. Regarding this additional heat storage gas volume as part of the regenerator, the distribution ratio was increased from $r_d = 1.21$ to $r_d = 4.5$. The calculated ideal value is $r_d = 4.7$! Using the results of Fig. 10, the model predicts an increase of the shaft power of about 2.9 times, which is 7.4% higher than the observed value. That is mainly due to the neglected opposite temperature distribution in the heat storage material placed at the hot side of the pulse tube.

The found dependency of the engine performance on the distribution ratio results from the working principle of the pulse tube engine. This is based on a broken thermodynamic symmetry between pulse tube and regenerator. Thermodynamic asymmetry is achieved by the set temperature distribution and the opposed heat transfer conditions in both components. At low distribution ratios the nearly adiabatic space (pulse tube) is large compared to the nearly isothermal space (regenerator). At high distribution ratios the inverse situation takes place. In both cases the thermodynamic symmetry is only slightly broken. For the found distribution ratio at maximum gross efficiency, the thermodynamic asymmetry is below its maximal value due to the influence of the heat transfer in the pulse tube. The maximum gross power is reached for a three times larger regenerator space, which should have exceeded the maximal value of thermodynamic asymmetry. In this case the mass flux crossing the hot heat exchanger is higher, which increases the amount of heat transferred to the working gas and thus the heat converted into expansion work.

To find the distribution ratio which maximizes the thermodynamic asymmetry, the pulse tube was regarded as ideal adiabatic and the calculation of the process was repeated for distribution ratios between 0.2 and 12. The adiabatic gross power as well as the adiabatic gross efficiency are also shown in Fig. 10. For values of

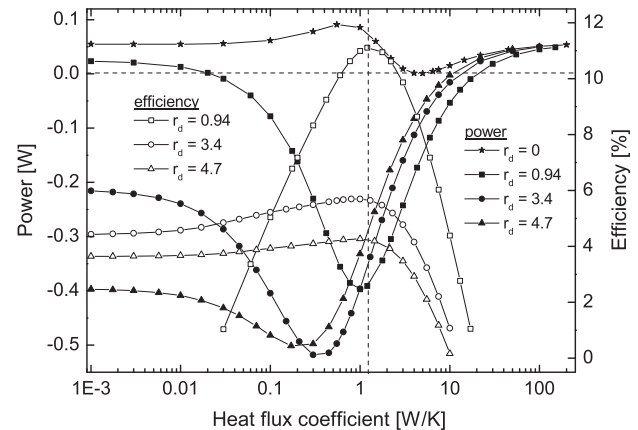


Fig. 11. Influence of pulse tube heat transfer on the engine performance.

$r_d < 2$, the adiabatic gross power is positive and the engine does not function. For values of $r_d > 2$, the thermodynamic asymmetry is high enough that a sufficient amount of heat energy can be converted into expansion work and provide a power output. The maximum adiabatic gross efficiency is reached at $r_d = 3.4$. This value will be regarded as the distribution ratio which maximizes the thermodynamic asymmetry of this engine. For smaller distribution ratios the adiabatic gross efficiency comes to zero and converges for higher distribution ratios to the gross efficiency of the non-adiabatic case. The same happens with the adiabatic gross power, which has its maximum at $r_d = 6$ and converges for higher values to the gross power of the non-adiabatic case. The reason of this behavior is the decreasing amount of heat transferred in the pulse tube.

5.4. Heat transfer conditions in the pulse tube

As discussed in Fig. 8, in a non-adiabatic pulse tube, heat is added to the working gas at the end of the compression and is released from it at the end of the expansion. This phenomena is called "thermal lag effect" [7,9]. To study the influence of this effect, the heat flux coefficient in the pulse tube was varied between adiabatic ($\gamma_{pt} = 1e^{-3}$ W/K) and isothermal ($\gamma_{pt} = 200$ W/K) conditions. The calculations were done for the case of a pulse tube only ($r_d = 0$), which is equal to perfect thermodynamic symmetry, for the distribution ratio of Hamaguchi et al. [8] ($r_d = 0.94$), for the distribution ratio of $r_d = 3.4$, which is regarded as the configuration of maximum thermodynamic asymmetry and for the distribution ratio of $r_d = 4.7$.

Fig. 11 shows the calculated gross powers and gross efficiencies for all four cases. The engine does not produce any power output for a pulse tube only ($r_d = 0$, no regenerator). In contrast to the regenerator of the Stirling engine, the regenerator of the pulse tube engine is essential for its functioning. In all other cases, the pulse tube engine is providing a power output, which have maximum values at specific heat flux coefficients. For higher distribution ratios, the heat flux coefficients which maximize the gross power decrease. The highest gross power of about -0.5 W is achieved for the conditions of maximum thermodynamic asymmetry ($r_d = 3.4$) at a heat flux coefficient of about 0.3 W/K. For the gross efficiency the heat flux in the pulse tube plays an important role. The maximum gross efficiency decreases for higher distribution ratios while the optimal heat flux coefficient for maximum gross efficiency remains almost constant. The highest gross efficiency of about 11% is achieved for the conditions of Hamaguchi et al. [8] ($r_d = 0.94$) at a heat flux coefficient of about 1.21 W/K. Thus, the pulse tube is acting as a regenerator, which takes on heat at

the end of the expansion and gives off heat at the end of the compression. The difference to the regenerator of the Stirling engine is that an optimal pulse tube has the property of a specific, optimal amount of heat to be exchanged between the pulse tube material and the working gas. At poor heat transfer conditions the pulse tube loses the regenerative property and at rich heat transfer conditions the thermodynamic asymmetry decreases. Therefore, the thermal lag effect increases the efficiency of the pulse tube engine, but is not responsible for its functioning. The underlying working principle is based on a broken thermodynamic symmetry.

6. Conclusions

The pulse tube engine is a simply structured heat engine with a considerable potential for waste heat utilization. With the developed simulation model a deeper understanding of the pulse tube engine process could be acquired. The most important results of this study are summarized.

6.1. Pulse tube engine process

- (1) Heat supply from the hot heat exchanger mostly occurs during the second half of the compression. During the expansion almost no heat is delivered by the hot heat exchanger.
- (2) During compression the regenerator material absorbs compression heat and the heat added by the hot heat exchanger. During expansion the regenerator material completely releases the entire heat.
- (3) In the pulse tube, heat input occurs towards the end of the compression phase and heat output occurs towards the end of the expansion phase.
- (4) Except for the pulse tube, all energy fluxes come to a standstill at the turning points of the piston.
- (5) In a cylinder with poor heat transfer the working gas is expanded below the cold heat exchanger temperature, thus making the cold heat exchanger regenerative as well.
- (6) During expansion heat is released by the cold heat exchanger. Thus, the pressure drops below ambient pressure, which decelerates the second phase of the expansion and accelerates the first phase of the compression due to the resulting force on the piston. This enables a free piston operation of the engine.

6.2. Design parameters

- (1) The compression ratio influences the performance of the engine significantly. Depending on the temperature of the hot heat exchanger, the compression ratio has optimal values for power output and efficiency. The optimal compression ratio of maximum efficiency is slightly higher than the optimal compression ratio of maximum power output.
- (2) The underlying working principle of the pulse tube engine is a broken thermodynamic symmetry. Thermodynamic asymmetry results from the fluidic linkage of an adiabatic space (pulse tube) with an isothermal space (regenerator) having the depicted temperature distribution. An engine having a perfect adiabatic pulse tube can provide a power output. The regenerator is essential for the functioning of the pulse tube engine.

- (3) The thermodynamic asymmetry can be maximized by the correct ratio between pulse tube volume and regenerator volume. The maximum power output is reached for more regenerator volume and the maximum efficiency for more pulse tube volume.
- (4) The thermal lag effect in the pulse tube supports the process and increases the efficiency of the engine. Maximum efficiency is obtained for a specific amount of heat transferred between pulse tube material and working gas. The pulse tube has thus a regenerative property, which is bound on well adjusted heat transfer conditions.

The developed simulation model is suitable for the analysis and dimensioning of the pulse tube engine. It allows for an investigation of the underlying thermodynamic process as well as for a study of design and operational parameters.

Acknowledgments

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